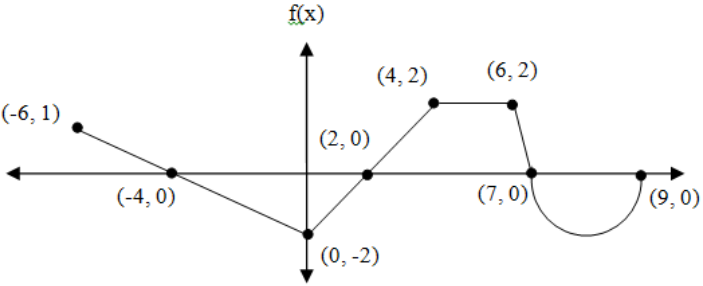


<p>Alan Tupaj  Vista Murrieta High School  Website: <a href="http://www.vmhs.net">www.vmhs.net</a>  (Click on "Teachers" then "Alan Tupaj")</p>	<p>Integration as Area  AP Readiness Session 6 - February    Answers to examples posted on my website</p>
<p><b>General Problem Steps</b></p>	<p><b>Examples</b></p>
<p>Given the graph of a function, find the area under the function over a given interval. (same as the definite integral of the function over the interval)  (must be linear or circular to use geometry)</p> <ul style="list-style-type: none"> <li>• Divide up the given interval into geometric shapes</li> <li>• Area below the x-axis is considered negative</li> <li>• If limits of integration are left to right (low to high), then each area changes sign</li> <li>• If evaluating a function defined as the integral of the given function from <math>x</math> to <math>a</math>, then you must add the initial condition given at <math>F(a)</math></li> </ul>	 <p>1. Find <math>\int_0^2 f(x)dx =</math>                      2. Find <math>\int_0^9 f(x)dx =</math></p> <p>3. Find <math>\int_2^{-6} f(x)dx =</math>                      4. Find <math>\int_8^6 f(x)dx =</math></p> <p>Given the graph of <math>f'(x)</math> and <math>f(0) = 3</math></p> <p>5. Find <math>f(-6) =</math>                              6. Find <math>f(6) =</math></p>
<p>Given a function, find the area under the graph over a given interval</p> <ul style="list-style-type: none"> <li>• Evaluate the definite integral of the function over the interval.</li> </ul>	<p>7. Find the area under the curve <math>f(x) = -x^2 + 6x - 3</math> on the interval <math>(1, 2)</math>.</p>
<p>Given a region defined by two functions, <math>f(x)</math> and <math>g(x)</math>, find the area of the region.</p> <ul style="list-style-type: none"> <li>• If <math>f(x) \geq g(x)</math>, then  <math display="block">\text{Area} = \int_a^b (f(x) - g(x))dx</math></li> <li>• If the interval from <math>b</math> to <math>a</math> is not given, then <math>b</math> and <math>a</math> are the points of intersection of <math>f(x)</math> and <math>g(x)</math></li> <li>• Find points of intersection by setting <math>f(x) = g(x)</math> and solving for <math>x</math></li> </ul>	<p>8. Find the area enclosed by <math>f(x) = 5 - x^2</math> and <math>g(x) = x - 7</math></p>

<p>Given a region defined by two functions, <math>f(x)</math> and <math>g(x)</math>, with more than two points of intersection, find the area of the region.</p> <ul style="list-style-type: none"> <li>• Find points of intersection by setting <math>f(x) = g(x)</math> and solving for <math>x</math></li> <li>• Determine which function is greater over each interval</li> <li>• Split into two integrals</li> </ul> $A = \int_c^b (f(x) - g(x))dx + \int_a^c (g(x) - f(x))dx$ <p>Where <math>a, b</math>, and <math>c</math> are points of intersection, <math>c</math> is between <math>a</math> and <math>b</math>, <math>f(x) \geq g(x)</math> between <math>b</math> and <math>c</math>, and <math>g(x) \geq f(x)</math> between <math>c</math> and <math>a</math></p>	<p>9. Find but do not evaluate an integral to represent the area enclosed by <math>f(x) = x^3 - 2x^2</math> and <math>g(x) = 2x^2 - 3x</math></p>
<p>Given a region defined by two relations where <math>x =</math> some expression of <math>y</math>, the area can be determined by an integral in the <math>y</math>-direction.</p> $\text{Area} = \int_c^d (f(y) - g(y))dy$ <p>Where <math>c</math> and <math>d</math> are the <math>y</math>-coordinates of the points of intersection and <math>f(y) \geq g(y)</math> (graph of <math>f(y)</math> is to the right of <math>g(y)</math>)</p>	<p>10. Find but do not evaluate an integral to represent the area enclosed by the graphs of <math>x = 3 - y^2</math> and <math>x = y + 1</math></p>
<p>Given a region defined by multiple boundaries, find the area.</p> <ul style="list-style-type: none"> <li>• Determine all intersecting points of the boundaries of the region</li> <li>• If necessary, split the region into multiple integrals</li> </ul>	<p>11. Find but do not evaluate an integral to represent the area enclosed by:  <math>y = 2\sqrt{x-1} - 3</math>, <math>y = -2x + 11</math>, and the <math>x</math>-axis</p>